Methods of Experimental Particle Physics

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Lecture #6
From Last Time...

- We have learnt about weak interaction
  - Responsible for radioactive decays
  - Also responsible for muon decay
  - And requires neutrino – a very light neutral particle
- Studied the old Fermi model with 4-fermion contact interaction:
  - Found that it has very bad divergences like \( \sigma \sim G_F^2 E^2 \)
- We offered a solution by introducing W bosons, in which case
  \[ \sigma \sim G_F^2 E^2 \rightarrow \frac{g^4}{(E^2 - M_W^2)^2} E^2 \]
  - We also made an analogy between a fermion (electron or neutrino) interacting with a W and rotations in SU(2)
    - Interaction “rotates” the fermion state \( (e) \), e.g. \( \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \)
    \( \begin{pmatrix} 1 \\ 0 \end{pmatrix} \) \( \begin{pmatrix} 0 \end{pmatrix} \)
  - For this to work, W’s will correspond to Pauli’s x- an y-matrices, but there is the third Pauli matrix – another boson?
    - Yes, the Z boson!
W and Z Boson Discoveries at CERN

• First evidence for Z bosons from neutrino scattering using Gargamelle bubble chamber
  • Sudden movement of electrons

\[ \nu \rightarrow Z^0 \rightarrow \nu \]

• Discovery of W boson and a very convincing confirmation of Z by UA1/UA2 from SPS (Super Proton Synchrotron)
  • 1981-1983
  • UA="Underground Area"
  • 400 GeV proton-antiproton beams
One issue is that W’s only interact with left-handed parts of fermion’s wave function

- We know how to get those mathematically

\[ \psi_L = \frac{1 - \gamma^5}{2} \psi, \quad \psi_R = \frac{1 + \gamma^5}{2} \psi \]

- We even can perhaps write down the part in the lagrangian for this kind of interaction

\[ \mathcal{L} = \bar{\psi} \left( \frac{1 + \gamma^5}{2} \right) \gamma^\mu (i \partial_\mu - g \frac{1}{2} \tau^\mu W_\mu (1 - \gamma^5) \psi \]

- Where Pauli matrices \( \tau \) act on vectors \( \psi \), which are 2-d vectors \( (e)_i^j \) containing wave-functions of lepton and its neutrino, vector W stands for \( W^+, W^- \) and Z bosons, each going with its own Pauli matrix
  - We skipped the fermion mass term for now

- But now we don’t have right-handed fermions
Right-Handed

• Some problems:
  • The kinetic term only includes left-handed fermions, what do you do with the right handed ones?
    • We want to also describe QED, which doesn’t care about left-handed or right handed fermions, so we need to put them back in
      • Add another piece by hand with $d_m$ just for the right handed fermions?
    • Also from experiment we know that $Z$ couples to both right-handed and left-handed fermions (even though not equally)
      • We need to put them back in, but how do you avoid having them couple to $W$’s?
  • One solution is separate left-handed and right-handed fermions into different representations
Representations of Groups

• SU(2) describes rotations in the complex 2-dimensional space
  • We have separated left-handed parts of fermion wave functions and put them into this doublet form \( (e^i) \)
  • 2-dimensional vectors are rotated using Pauli matrices (generators of SU(2))

• Do you know any other “things” other than vectors than can live in multi-dimensional space?

• Scalars! We can consider right-handed parts of fermions to be scalars (they won’t rotate as you do transformations, they just remain as they are
Left- and Right-Handed Terms

• Can technically add a new piece:

\[
\mathcal{L} = \overline{e}(\frac{1+\gamma^5}{2})\gamma^\mu(i\partial_\mu - g\frac{1}{2}\tau W_\mu\left(\frac{1-\gamma^5}{2}\right)e) + \bar{\nu}(\frac{1-\gamma^5}{2})\gamma^\mu(i\partial_\mu - g\frac{1}{2}W'_\mu\left(\frac{1+\gamma^5}{2}\right)e +
\]

\[
\overline{\nu}(\frac{1-\gamma^5}{2})\gamma^\mu(i\partial_\mu - g\frac{1}{2}W'_\mu\left(\frac{1+\gamma^5}{2}\right)v
\]

• I reinstated the kinetic term

• And also allowed some new potential interaction with a new W’ field, e.g. W_3 and W’ could potentially be the Z boson that I know should couple to both left and right-handed fields

  • Let’s remember this!

• We can also use new notations

  • \(E_L = (\nu e)_L = (\frac{1-\gamma^5}{2})e\) and \(Q_L = (\nu u)_L\) for quarks

  • \(e_R = \left(\frac{1+\gamma^5}{2}\right)e\) and the same for neutrinos
The Lagrangian again

\[ \mathcal{L} = \overline{E_L} \gamma^\mu (i \partial_\mu - g \frac{1}{2} \tau W_\mu) E_L + \overline{e_R} \gamma^\mu (i \partial_\mu - g \frac{1}{2} W'_\mu) e_R + \overline{\nu_R} \gamma^\mu (i \partial_\mu - g \frac{1}{2} W'_\mu) \nu_R \]

- An interesting thing is that you can use \( W \) (couples to left-handed fermions only) and \( W' \) (couples to right handed fermions only) to build a physical photon and the \( Z \)-boson
- And re-write (sorry about different notations)

\[
\mathcal{L}_{EW} = \sum \overline{\psi} \gamma^\mu \left( i \partial_\mu - g' \frac{1}{2} Y_W B_\mu - g \frac{1}{2} \tau L \tilde{W}_\mu \right) \psi
\]
  - Subscript \( L \) means Pauli matrices act only on left-handed fields
  - Then \( Z = aB + bW_3 \) and \( \gamma = cB + dW_3 \) (you need to pick parameters to preserve unitarity)
- This is a step towards electroweak unification!
  - Physical bosons are a mixture of true EM and Weak interaction bosons
Review

- W bosons work only on left-handed components of all fermions
  - They change their type and charge (e→nu, u→d)
- Z bosons work on both left-handed and right-handed components, but not necessarily with the same strength
  - Can’t change charge, but can couple to neutrinos
- Photons work on both left-handed and right-handed fermions
  - But only on charged ones, e.g. it won’t interact with neutrinos
Are we good?

- No, because we ignored two problems:
  - You can’t have mass for an electron in our new theory
    - Mass term in our notations should look like
      \[ \mathcal{L} = -m_e (\bar{e}_L e_R + \bar{e}_R e_L) \] but you can’t write this because left and right handed parts live in different worlds
    - If you multiply a vector by a scalar, you get a vector, while the lagrangian must be a scalar
  - Gauge bosons can’t have masses
    - Photon is okay, but W+/− and Z have large masses
    - They are generators, so you can’t also add them as physical “free” particles with a corresponding mass term
Fermion Masses

- Forget for now about gauge boson masses
- Let’s see if there is a way to make fermion masses not zero without adding a mass term explicitly
- Say we add a new scalar field $\phi$: and allow it to couple to fermions:

$$\Delta \mathcal{L}_e = -\lambda_e \overline{E}_L \cdot \phi e_R + h.c.$$ 

- What if this field can have a non-zero VEV $v$?

$$\Delta \mathcal{L}_e = -\frac{1}{\sqrt{2}} \lambda_e v \overline{\epsilon}_L e_R + h.c. + \cdots$$

- Then you effectively “generate” fermion mass to be

$$m_e = \frac{1}{\sqrt{2}} \lambda_e v.$$
Higgs Potential

• Write a lagrangian for the new scalar field as follows:

\[ \mathcal{L} = \left| \partial \phi \right|^2 + \mu^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2. \]

• Let’s say this new field has VEV of \( v \), expand:

\[ \phi(x) = U(x) \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix} \]

• The minimum of potential energy occurs at

\[ v = \left( \frac{\mu^2}{\lambda} \right)^{1/2}. \]
Higgs Boson

• Now expand the lagrangian around \( v \):

\[
\mathcal{L} = \left| D_\mu \phi \right|^2 + \mu^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2.
\]

• You will get the following:

\[
\mathcal{L}_V = -\mu^2 h^2 - \lambda v h^3 - \frac{1}{4} \lambda h^4
\]

\[= -\frac{1}{2} m_h^2 h^2 - \sqrt{\frac{\lambda}{2}} m_h h^3 - \frac{1}{4} \lambda h^4.\]

• The first term is the new boson’s mass!

• It can be expressed as:

\[m_h = \sqrt{2} \mu^2 = \sqrt{\frac{\lambda}{2}} v.\]

• If you re-write above lagrangian (remember you use covariant derivative, which includes W and Z in it)

\[
\mathcal{L}_K = \frac{1}{2} (\partial_\mu h)^2 + \left[ m_W^2 W^\mu W_\mu + \frac{1}{2} m_Z^2 Z^\mu Z_\mu \right] \cdot \left( 1 + \frac{h}{v} \right)^2,
\]

• This new field can generate W and Z masses
• Review of the Standard Model
  • Basic phenomenology
    • Include relationship of W and Z masses as they are derived from the lagrangian we wrote today
    • Three generations
  • Experimental tests
    • Masses of particles, widths etc.
    • Interaction strengths

• We will conclude with the theoretical introduction
  • Will briefly return to it later when we discuss QCD